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rent, having thus yielded to the first onset, collected its strength to force its opponent back. On the 9th there was a lull, followed by a north wind lasting up to the 11th, during which time the barometer rose 7·46 lines, and the thermometer fell 15° 4. On the evening of the 11th there was a fresh lull; and on the 12th the equatorial current set in again, as is shown by the wind getting round to S. W., the thermometer rising above Zero, and the barometer falling 6·06 lines.'

"Yours very sincerely,
"ROBERT H. SCOTT.

"*Rev. Professor Haughton.*"

PROFESSOR HENNESSY, F. R. S., read the following paper:—

ON CLAIRAUT'S THEOREM.

LAPLACE has shown that this theorem follows whatever may be the density of the interior parts of the earth, provided it consists of similar concentric strata, and that the form of the outer stratum is ellipsoidal. In the "Philosophical Transactions" for 1826, Mr. Airy (the present Astronomer Royal of England) has presented an equivalent result; more recently, Professor Stokes has shown that we can deduce the law of variation of terrestrial gravity without any hypothesis whatsoever as to the earth's interior structure. He assumes merely that its surface is spheroidal, and that the equation of fluid equilibrium holds good at that surface. In vol. vi. of the "Cambridge Mathematical Journal," Professor Haughton presented a demonstration, founded upon the same assumptions as those of Professor Stokes, and in which he uses certain propositions relative to attractions which had been enunciated by Gauss and Mac Cullagh. While studying the labours of those mathematicians, it appeared to me that the question could be entirely divested of the hydrostatical character, and that Clairaut's theorem may be directly deduced from the equations to the normal of any closed surface, without any considerations as to the physical condition of the matter forming that surface. Thus every surface concentric with the earth, and perpendicular to gravity, will possess the property of exhibiting this relation in the intensity of gravity at its various points.

Let X, Y, Z represent the components parallel to the rectangular axes of the forces by which a point is retained at rest on a given surface whose equation is $L = 0$. Then from the equations of the normal we have

$$Y\frac{dL}{dx} - X\frac{dL}{dy} = 0, \quad Z\frac{dL}{dx} - X\frac{dL}{dz} = 0,$$

when the resultant of these forces is perpendicular to the given surface. If we represent by V the potential of the earth on the particle in question, by w the angular velocity of rotation, we have

$$X = \frac{dV}{dx} + w^2x, \; Y = \frac{dV}{dy} + w^2y, \; Z = \frac{dV}{dx},$$

and the above equations become

$$\frac{dV}{dy}\frac{dL}{dx} - \frac{dV}{dx}\frac{dL}{dy} = w^2\left(x\frac{dL}{dy} - y\frac{dL}{dx}\right)$$

$$\frac{dV}{dz}\frac{dL}{dx} - \frac{dV}{dx}\frac{dL}{dz} = w^2\, x\, \frac{dL}{dz}.$$

If, in conformity with General Schubert's* recent determinations, we assume the earth's surface to be that of an ellipsoid, with three unequal axes, we should substitute for L

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0,$$

or

$$\frac{dL}{dx} = \frac{2x}{a^2},\ \frac{dL}{dy} = \frac{2y}{b^2},\ \frac{dL}{dz} = \frac{2z}{c^2};$$

whence we have

$$b^2x\frac{dV}{dy} - a^2y\frac{dV}{dx} = w^2xy\,(a^2 - b^2),\ c^2x\frac{dV}{dz} - a^2z\frac{dV}{dx} = w^2a^2xz.$$

Each of these partial differential equations can be easily integrated, and the value of V, finally obtained, is equivalent to the equation of fluid equilibrium; or

$$V + \frac{w^2}{2}\,(x^2 + y^2) = C.$$

Let θ represent the complement of the latitude, and ϕ the longitude, counted from the meridian of the greatest axis, then $z = r\cos\theta$, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, and

$$V + \frac{r^2w^2}{2}\sin^2\theta = C.$$

In the case of an ellipsoid having the ellipticity e, we have, neglecting small terms,

$$r = a\,(1 - e\cos^2\theta).$$

From these equations, and from the properties of Laplace's functions into which V can be expanded, an expression can be obtained of the same kind as that deduced by Professor Stokes from his own and Gauss' theorems relative to attractions.

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* Mémoires de l'Académie Imperiale des Sciences de St. Petersbourg, VII^e serie, tome i.